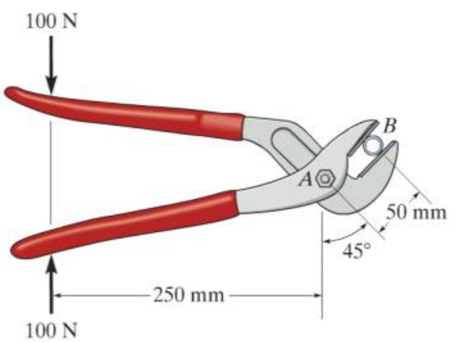
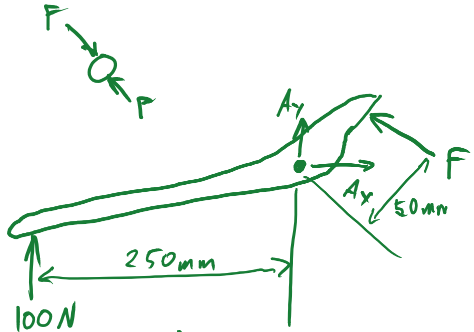
Frame and Machine Analysis

Sunday, April 9, 2017 9:44 PM



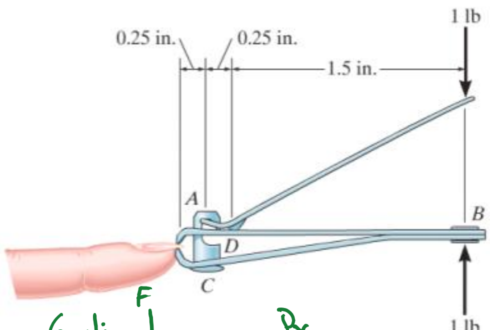
Before starting a problem, consider the easiest path to find your unknowns



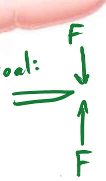
$(\Sigma M)_A = 0$

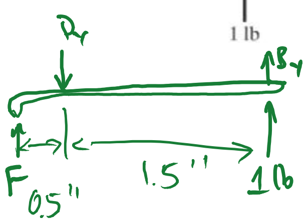
$\Rightarrow -250\,mm \cdot 100\,N + 50\,mm \cdot F = 0$

$\boxed{F = \frac{250}{50} \cdot 100\,N = 500\,N}$



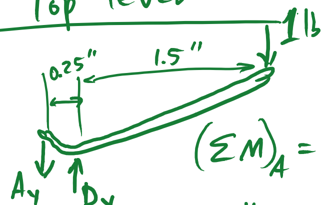
Goal:





if we can find D_y, we can take $(\Sigma M)_B = 0$ to solve F

Top lever



$(\Sigma M)_A = 0$

$0.25'' \cdot D_y - 1.75'' (1\,lb) = 0$

$D_y = \frac{1.75}{0.25}(1\,lb) = 7\,lb$

Middle lever

$(\Sigma M)_B = 0$

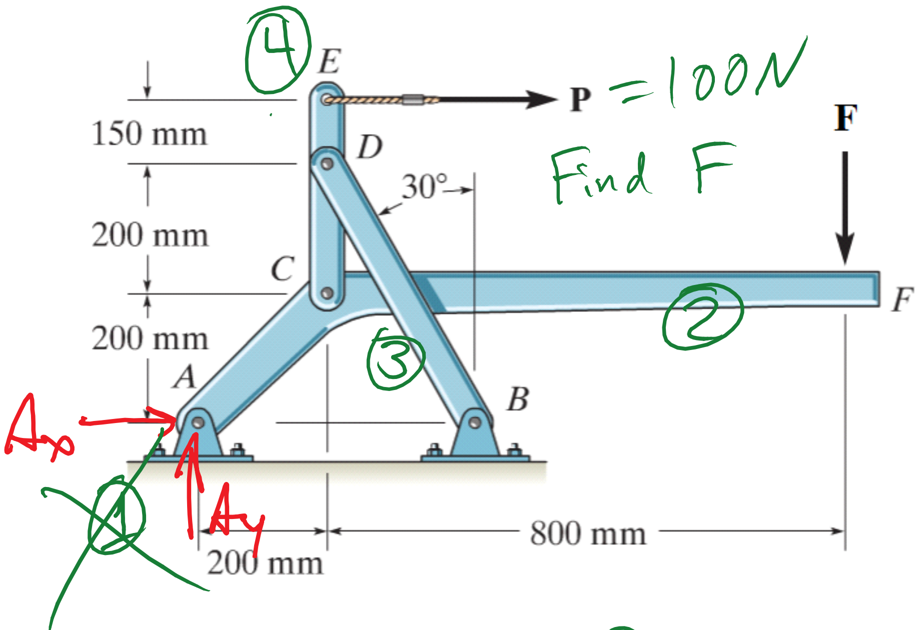
$1.5 \cdot D_y - 2 \cdot F = 0$

$\boxed{F = \frac{1.5}{2} \cdot D_y = \frac{3}{4}(7\,lb) = 5.25\,lb}$

$(\Sigma M)_B =$

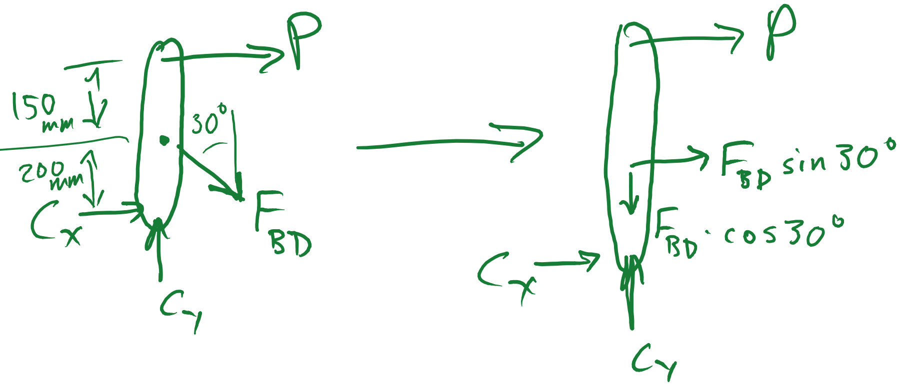
$1.5\cdot D_y - 2\cdot F = 0$

$$\boxed{F = \frac{1.5}{2}\cdot D_y = \frac{3}{4}(7\text{ lb}) = 5.25\text{ lb}}$$



Start with 4

$(\Sigma M)_C = 0$



$(\Sigma M)_C = 0$

$-(200\text{ mm})F_{BD}\cdot\sin 30° - (350\text{ mm})P = 0$

$$\boxed{F_{BD} = -\frac{350}{200}\frac{P}{\sin 30°} = -3.5\cdot P}$$

solve for C_x & C_y. $\Sigma F_y = 0 \Rightarrow C_y - F_{BD}\cdot\cos 30° = 0$

$$\boxed{C_y = F_{BD}\cdot\cos 30° = -3.5\cdot P\cdot\frac{\sqrt{3}}{2}}$$

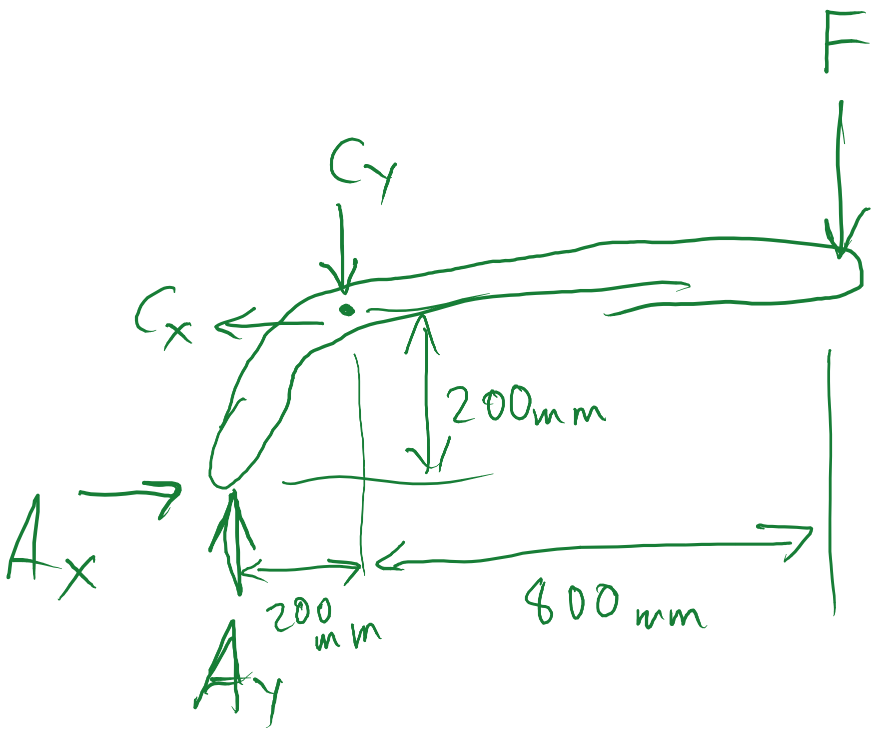
$$\Sigma F_x = 0 \Rightarrow P + F_{BD} \cdot \sin 30° + C_x = 0$$

$$C_x = -P - F_{BD} \cdot \frac{1}{2}$$

$$C_x = -P - (-3.5 \cdot P) \cdot \frac{1}{2}$$

$$= -P + 1.75 P$$

$$\boxed{C_x = 0.75 \cdot P}$$



$$(\Sigma M)_A = 0$$

$$200 C_x - 200 C_y - 1000 F = 0$$

$$-1000 F = 200 (C_y - C_x)$$

$$F = \frac{C_x - C_y}{}$$

$$F = \frac{C_x - C_y}{5}$$

$$= \frac{0.75 \cdot P - \left(-3.5 \frac{\sqrt{3}}{2} \cdot P\right)}{5} = \boxed{0.756 \cdot P}$$

Center of Gravity

Monday, April 3, 2017 1:00 AM

Determine the location of the center of gravity of the three-wheeler. The location of the center of gravity of each component and its weight are tabulated in the figure. If the three-wheeler is symmetrical with respect to the x-y plane, determine the normal reaction each of its wheels exerts on the ground.

	W	$\bar{x}$ [ft]	$\bar{y}$ [ft]
1. Rear wheels	18 lb	4.5	1.3
2. Mechanical components	85 lb	2.3	1.5
3. Frame	120 lb	3.1	2
4. Front wheel	8 lb	0	1

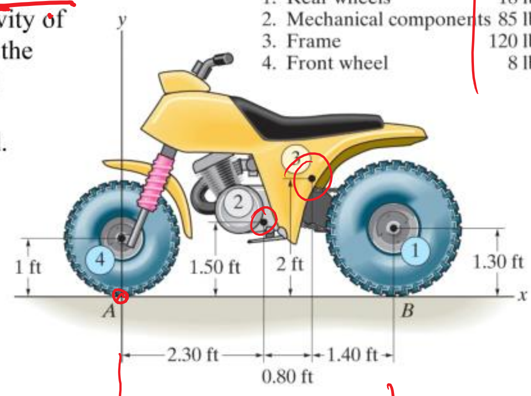
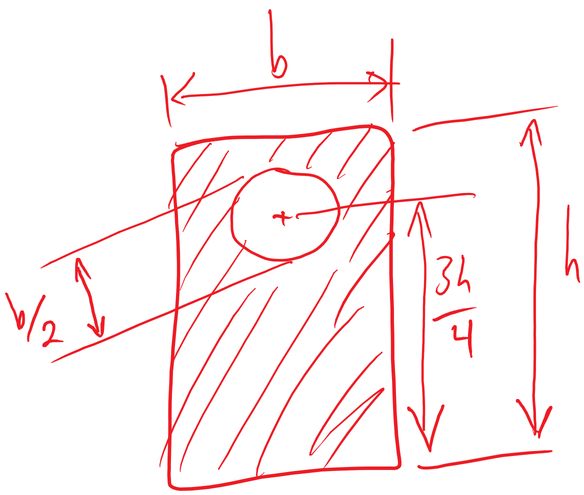


Figure: 09_P077

$$\bar{X} = \frac{\bar{x}_1 W_1 + \bar{x}_2 W_2 + \bar{x}_3 W_3 + \bar{x}_4 W_4}{W_1 + W_2 + W_3 + W_4}$$



Find $\bar{Y}$

$$\bar{Y} = \frac{\bar{Y}_\square A_\square - \bar{Y}_o A_o}{A_\square - A_o}$$

Area of circle $= \frac{\pi}{4} d^2$

$d = b/2$

$\bar{Y}_\square = h/2 \quad A_\square = b \cdot h$

$\bar{Y}_o = \frac{3h}{4} \quad A_o = \frac{\pi}{4}\left(\frac{b}{2}\right)^2$

$$\bar{Y} = \frac{\frac{h}{2}(bh) - \frac{3h}{4}\left(\frac{\pi b^2}{16}\right)}{\frac{16bh}{16} - \frac{\pi b^2}{16}} = \frac{\frac{bh^2}{2} - \frac{3\pi b^2 h}{64}}{\frac{16bh - \pi b^2}{16}} \quad \text{etc.}$$

-16 16

Internal Forces: Shear and Bending Moment

Sunday, April 9, 2017 9:45 PM

$$\frac{dV}{dx} = w(x)$$

Slope of shear diagram = Distributed load intensity

$$\Delta V = \int w(x)\,dx$$

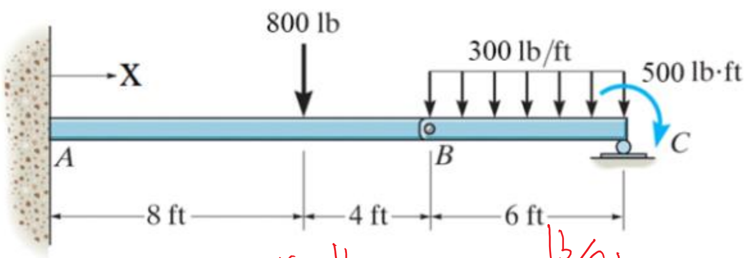
Change in shear = Area under loading curve

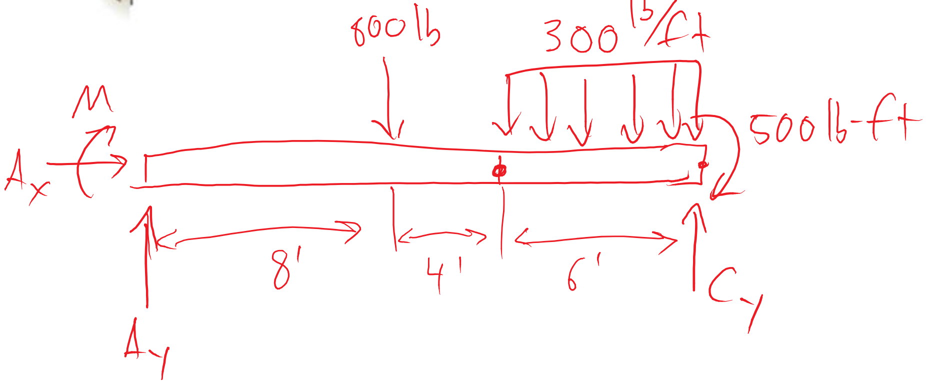
$$\frac{dM}{dx} = V$$

Slope of moment diagram = Shear

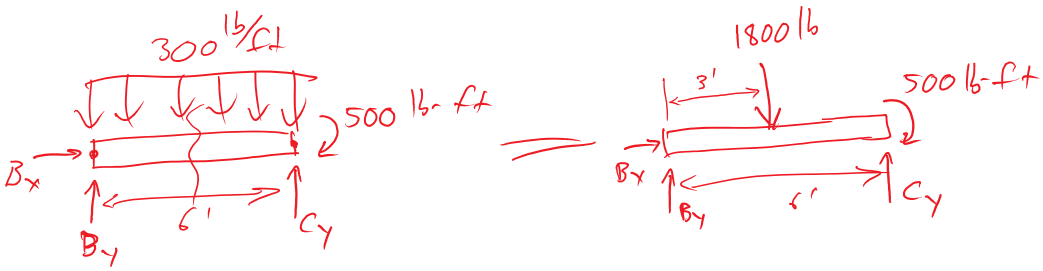
$$\Delta M = \int V\,dx$$

Change in moment = Area under shear diagram



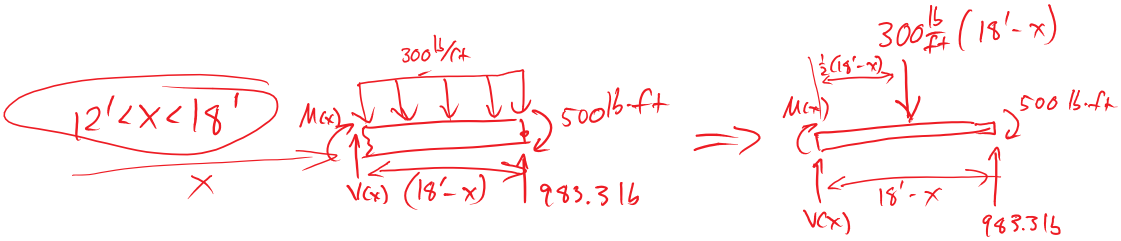


start w/ FBD & external equilibrium



$$(\Sigma M)_B = 0 \Rightarrow 6' \cdot C_y - 500\,\text{lb}\cdot\text{ft} - (3')(1800\,\text{lb}) = 0$$

$$\boxed{C_y = \frac{5400\,\text{ft}\cdot\text{lb} + 500\,\text{lb}\cdot\text{ft}}{6'} = 983.3\,\text{lb}}$$

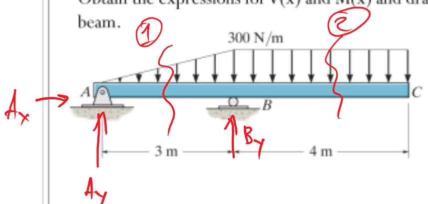


$$(\Sigma M)_x = 0$$

$$\Rightarrow -M(x) - \tfrac{1}{2}(18' - x)(300\,\tfrac{\text{lb}}{\text{ft}})(18' - x) - (500\,\text{lb-ft}) + (983.3\,\text{lb})(18' - x) = 0$$

solve for $M(x)$

Obtain the expressions for V(x) and M(x) and draw the shear and bending moment diagram for the beam.

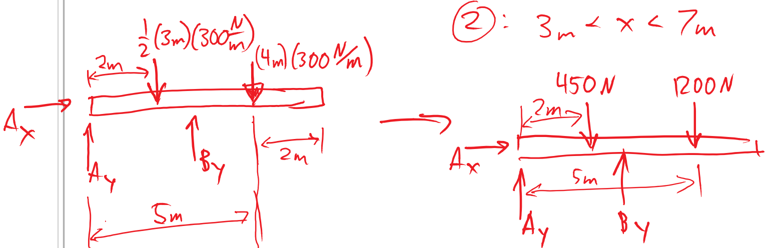


2 sections of the beam

Cut at x for

①: $0 < x < 3\,m$

②: $3\,m < x < 7\,m$



$$(\Sigma M)_A = 0 \Rightarrow -(2m)(450N) - (5m)(1200N) + (3m)B_y = 0$$

$$-900\,Nm - 6000\,N\cdot m = -3m \cdot B_y$$

$$B_y = \frac{6900\,N\cdot m}{3\,m} = 2300\,N$$

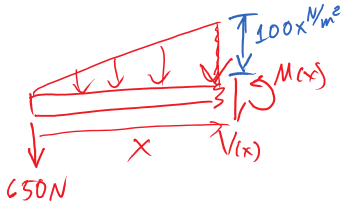
$$\Sigma F_x = 0 \Rightarrow A_x = 0$$

$$\Sigma F_y = 0 \Rightarrow A_y + B_y - 450N - 1200N = 0$$

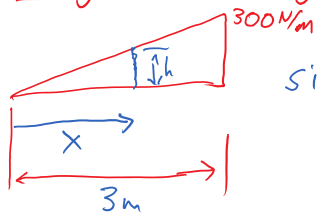
$$A_y = 1650N - B_y = (1650 - 2300)\,N$$

$A_y = -650\,N$

Cut @ 1 $\quad 0 < x < 3m$



height of triangle



similar triangles!

$$\frac{h}{x} = \frac{300\,N/m}{3m}$$

$$h(x) = \left(100\,\tfrac{N}{m^2}\right)\cdot x$$

$\Sigma F_y = 0$

$$\Rightarrow -V(x) - 650N - \tfrac{1}{2}\left(100x\cdot\tfrac{N}{m^2}\right)\cdot x = 0$$

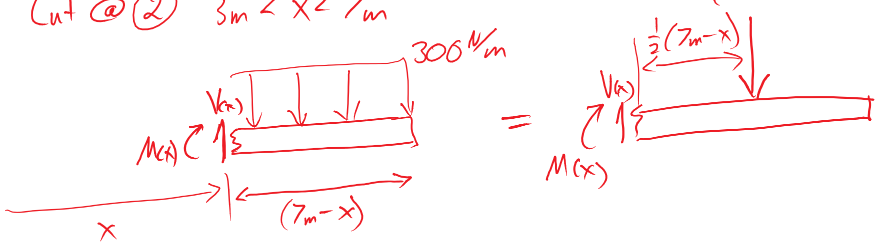
$$\boxed{V(x) = -\left(50\tfrac{N}{m^2}\right)x^2 - 650N}$$

$$(\Sigma M)_x = 0 \Rightarrow M(x) + (650N)\cdot x + \tfrac{x}{3}\left(\tfrac{1}{2}\cdot x\cdot 100\tfrac{N}{m^2}\cdot x\right) = 0$$

$$\boxed{M(x) = -x^3\cdot\left(\tfrac{100}{6}\,\tfrac{N}{m^2}\right) - (650N)\cdot x}$$

Shear & Bending moment in region 1

Cut @ 2 $\quad 3m < x < 7m$



$$\Sigma F_y = 0 \Rightarrow V(x) - \left(300\tfrac{N}{m}\right)(7m - x) = 0$$

$$\boxed{V(x) = 2100N - \left(300\tfrac{N}{m}\right)x}$$

Integrate: $\quad V(x) = \dfrac{dM}{dx}$

$$\int_x^{7m} V(x')\cdot dx' = \underbrace{\int_x^{7m} \frac{dM}{dx'}\,dx'}$$

2nd fundamental theorem of calculus

theorem of calculus

$$\underbrace{\qquad}_{M(x=7m)\,-\,M(x)}$$

$$M(7m) - M(x) = \int_x^{7m} V(x')\cdot dx'$$

$$M(x) = \underbrace{\cancel{M(7m)}}_{0} - \int_x^{7m} V(x')\cdot dx'$$

@ $x = 7m$ there is no couple

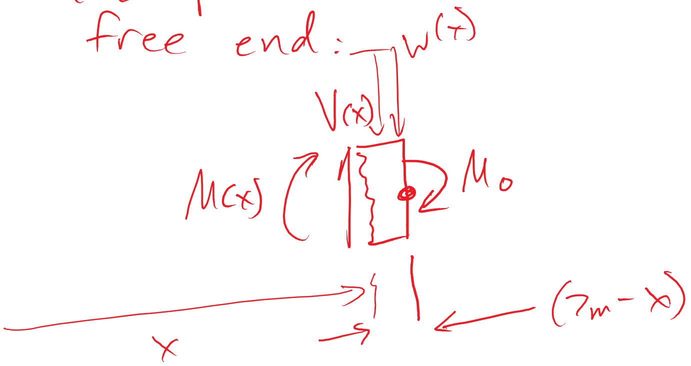
$$\Rightarrow M(7m) = 0$$

$$M(x) = -\int_x^{7m} \left(2100N - 300\tfrac{N}{m}\, x'\right)\cdot dx$$

$$M(x) = \left(2100N\cdot x' - \tfrac{1}{2}\left(300\tfrac{N}{m}\right)x'^2\right)\Big|_x^{7m}$$

$$M(x) = 2100N\,(7m - x) - \tfrac{1}{2}\left(300\tfrac{N}{m}\right)\left[(7m)^2 - x^2\right]$$

If there were a couple at the free end:



$$\left(\Sigma M\right)_x = 0$$

$$-M(x) - M_0 - \frac{w(x)\,(7m - x)^2}{2} = 0$$

take $x \to 7m$

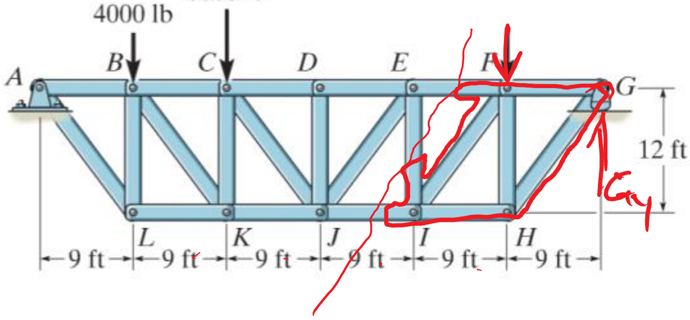
$$\Rightarrow -M(7m) - M_0 = 0$$

$$M(7m) = -M_0$$

Method of Sections

Sunday, April 9, 2017 9:46 PM

Determine the force in members EI and JI of the truss which serves to support the deck of a bridge. State if these members are in tension or compression.



1. External Equilibrium
2. Locate all zero-force members
3. Cut through no more that three unknown members
4. FBD of one side (take your pick)

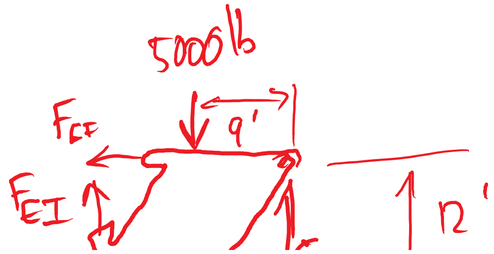
Solve for G_y

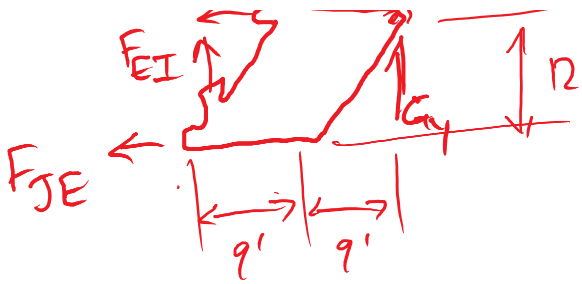
$$(\Sigma M)_A = 0$$

$$\Rightarrow -(9')(4000\,lb) - (18')(8000\,lb) - (45')(5000\,lb) + (54')G_y = 0$$

$$G_y = \frac{36000 + 144000 + 225000}{54}\,lbs = 7500\,lb$$

FBD of right-hand side of cut





$$\Sigma F_y = 0$$

$$G_y - 5000\,\text{lb} + F_{EI} = 0$$

$$F_{EI} = 5000\,\text{lb} - G_y$$

$$\boxed{F_{EI} = -2500\,\text{lbs}}$$ C

$$(\Sigma M)_G = 0$$

$$(9')(5000\,\text{lb}) - (18')F_{EI} - (12')F_{JE} = 0$$

$$45000\,\text{lb}\cdot\text{ft} - (18')(-2500\,\text{lb}) = 12' \cdot F_{JE}$$

$$F_{JE} = \frac{45000\,\text{lb}\cdot\text{ft} + 45000\,\text{lb}\cdot\text{ft}}{12'}$$

$$\boxed{F_{JE} = 7500\,\text{lb}}$$ T

Constraints and Reactions

Sunday, April 9, 2017 9:46 PM

TABLE 5–1 Supports for Rigid Bodies Subjected to Two-Dimensional Force Systems

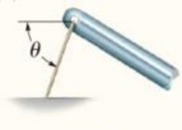
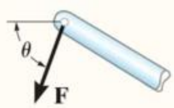
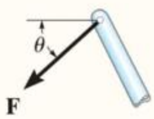
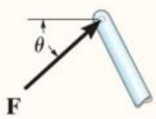
Types of Connection	Reaction	Number of Unknowns
(1)  cable		One unknown. The reaction is a tension force which acts away from the member in the direction of the cable.
(2)  weightless link	 or 	One unknown. The reaction is a force which acts along the axis of the link.
(3)  roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.

TABLE 5–1 Supports for Rigid Bodies Subjected to Two-Dimensional Force Systems

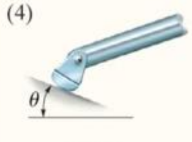
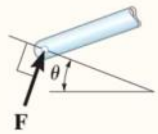
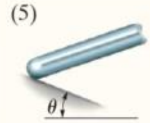
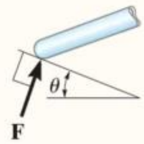
Types of Connection	Reaction	Number of Unknowns
(4)  rocker		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(5)  smooth contacting surface		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.

TABLE 5–1 Supports for Rigid Bodies Subjected to Two-Dimensional Force Systems

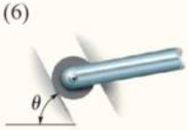
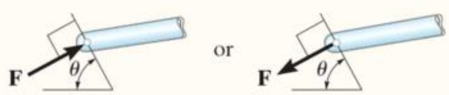
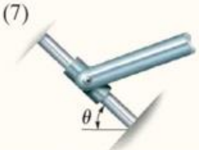
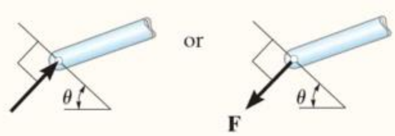
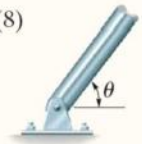
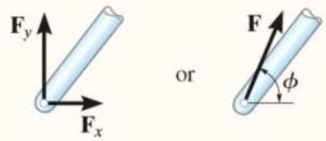
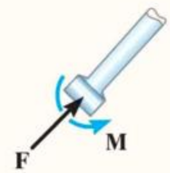
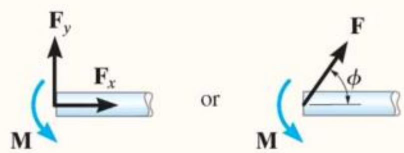
Types of Connection	Reaction	Number of Unknowns
(6)  roller or pin in confined smooth slot		One unknown. The reaction is a force which acts perpendicular to the slot.
(7)  member pin connected to collar on smooth rod		One unknown. The reaction is a force which acts perpendicular to the rod.

TABLE 5–1 Supports for Rigid Bodies Subjected to Two-Dimensional Force Systems

Types of Connection	Reaction	Number of Unknowns
(8)  smooth pin or hinge		Two unknowns. The reactions are two components of force, or the magnitude and direction ϕ of the resultant force. Note that ϕ and θ are not necessarily equal [usually not, unless the rod shown is a link as in (2)].
(9)  member fixed connected to collar on smooth rod		Two unknowns. The reactions are the couple moment and the force which acts perpendicular to the rod.
(10)  fixed support		Three unknowns. The reactions are the couple moment and the two force components, or the couple moment and the magnitude and direction ϕ of the resultant force.